MUMBAI UNIVERSITY

SEMESTER-1

ENGINEERING MECHANICS SOLVED PAPER-MAY 2017

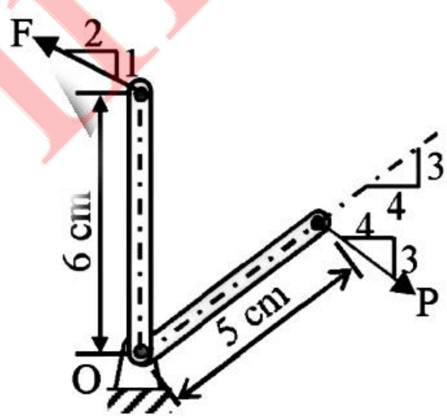
N.B:-(1)Question no.1 is compulsory.

(2)Attempt any 3 questions from remaining five questions.

(3)Assume suitable data if necessary,and mention the same clearly.

(4)Take g=9.81 m/s^2,unless otherwise specified.

Q.1(a) In the rocket arm shown in the figure the moment of 'F' about 'O' balances that P=250 N.Find F. (4 marks)

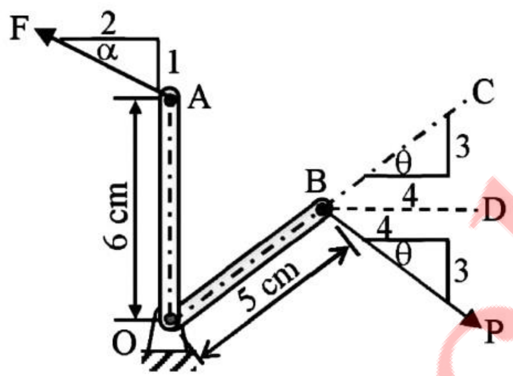


Solution :

Given : P = 250 N

To find : Magnitude of force F

Solution :



tan **α** = $\frac{1}{2}$

= 0.5

α = 26.5651°

tan θ = $\frac{DE}{AD} = \frac{DE}{BC} = \frac{3}{4}$ = 0.75

θ = 36.87°

∠CBD = ∠PBD = θ = 36.87°

∠CBP=2 θ = 2 x 36.87 = 73.74°

It is given that at O the moment of F about O balances the moment of P

Fcos α x OA = Psin2 θ x OB

Fcos26.5651 x 6 = 250sin 73.74 x 5

F=223.6068 N

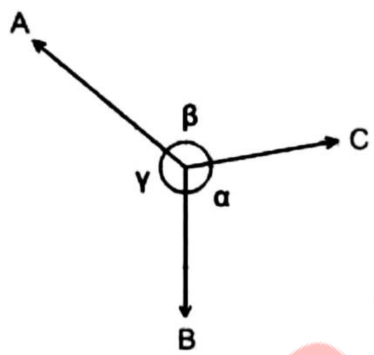
Magnitude of force F= 223.6068 N

Q.1(b) State Lami's theorem.

State the necessary condition for application of Lami's theorem. (4 marks)

Answer :

Lami's theorem states that if three forces acting at a point are in equilibrium, each force is proportional to the sine of the angle between the other two forces.



According to Lami's theorem, the particle shall be in equilibrium if :

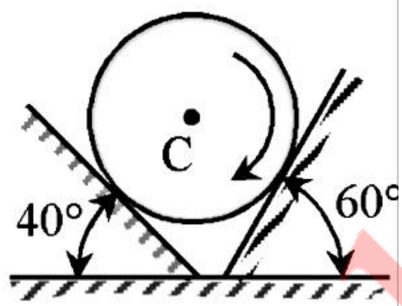
$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

The conditions of Lami's theorem are:

(1)Exact 3 forces must be acting on the body.

(2)All the forces should be either converging or diverging from the body.

Q.1(c)A homogeneous cylinder 3 m diameter and weighing 400 N is resting on two rough inclined surface's shown.If the angle of friction is 15º.Find couple C applied to the cylinder that will start it rotating clockwise. (4 marks)



Solution :

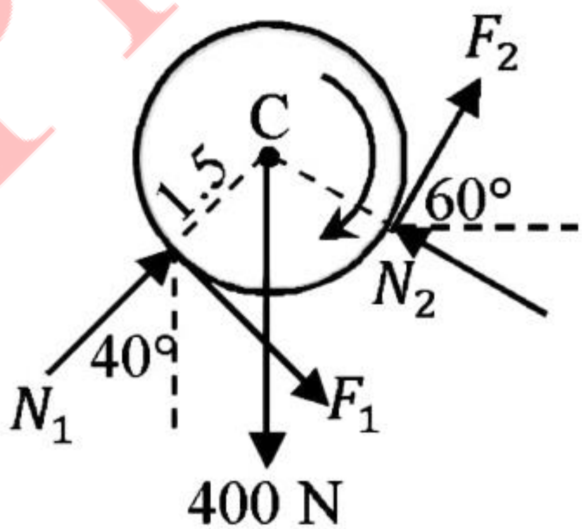
Given : Angle of friction is 15o

$\mu = \tan 15 = 0.2679$

Radius = 1.5 m

To find : Couple C

Solution:



$F_1=\mu N_1=0.2679N_1$ ………………….(1)

$F_2=\mu N_2=0.2679N_2$ ………..……….(2)

Assuming the body is in equilibrium

ΣFx=0

$F_1\cos40+N_1\sin40+F_2\cos60-N_2\sin60=0$

$N_1(0.2679\cos40 + \sin40)+N_2(0.2679\cos60-\sin60)=0$ …………(3)

ΣFy=0

$-F_1\sin40+N_1\cos40+F_2\sin60+N_2\cos60-400=0$

$N_1(-0.2679\sin40+\cos40)+N_2(0.2679\sin60+\cos60)=400$ ………..(4)

Solving (3) and (4)

$N_1=277.4197$ N and $N_2=321.3785$ N

Substituting N1 and N2 in (1 and 2)

$F_1=0.2679 \times 277.4197 = 74.3344$ N

$F_2=0.2679 \times 321.3785 = 86.1131$ N …….(5)

C is the couple required to rotate the cylinder clockwise

$C=F_1 \times r + F_2 \times r$

= 240.6712 Nm(clockwise) (r=1.5 m)(From 5)

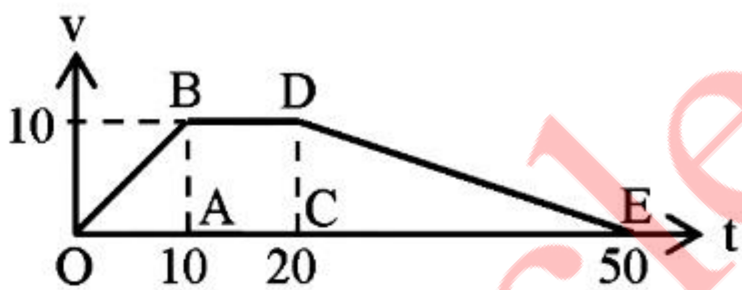
The couple C required to rotate the cylinder clockwise is 240.6712 Nm(clockwise)

Q.1(d) From (v-t) diagram find

(1) Distance travelled in 10 second.

(2) Total distance travelled in 50 second.

(3) Retardation (4 marks)



Solution:

We know that the area under v-t graph gives the distance travelled

DISTANCE TRAVELLED IN 0 TO 10 sec = A(△OAB)

$= \frac{1}{2}$ x OA x AB

$= \frac{1}{2}$ x 10 x 10

= 50 m

DISTANCE TRAVELLED IN 0 TO 50 sec = A(Trapezium OBDE)

$= \frac{1}{2}$ x (OE+BD) x AB

$= \frac{1}{2}$ x (50+10) x 10

= 300 m

CONSIDER THE MOTION FROM 20 sec TO 50 sec

We know that slope of v-t graph gives acceleration

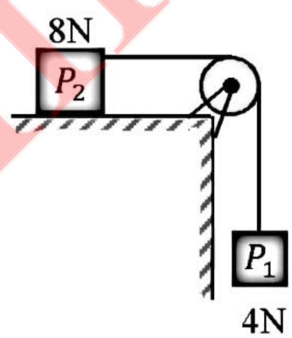
E=(50,0) and D=(20,10)

Slope of line DE= $\frac{0-10}{50-20}$ = $\frac{-1}{3}$ = -0.3333 m/s^2

Distance travelled by object in 10 sec = 50 m
Distance travelled by object in 50 sec = 300 m
Acceleration = - 0.3333 m/s2

Q1(e))Blocks P_1 and P_2 are connected by inextensible string.Find velocity of block P_1,if it falls by 0.6 m starting from rest.

The co-efficient of friction is 0.2.The pulley is frictionless. (4 marks)



Solution:

Given : P_1 falls by 0.6 m starting from rest

$\mu = 0.2$

To find : Velocity of block P_1

Solution :

Consider the motion of block P_2

Weight of motion $P_2 = 8$ N

Mass of $P_2 = \frac{8}{g}$

P2 has no vertical motion

$\Sigma F_y = 0$

$N_2 - 8 = 0$

$N_2 = 8$ N

$F_2 = \mu N_2$

$= 1.6$ N

Consider the horizontal motion

$\Sigma F_x = m_2 a$

$T - F_2 = m_2 a$

$T = 1.6 + \frac{8}{g}a$ ……………(1)

For block P_1
Weight of $P_1 = 4$ N

Mass of $P_1 = \frac{4}{g}$ ……….(2)

For downward motion

$\Sigma F_y = m_1 a$

$4 - T = m_1 a$

$4 - 1.6 - \frac{8}{g}a = \frac{4}{g}a$ (From 1 and 2)

$a = 1.962$ m/s^2

$v^2 = u^2 + 2as$

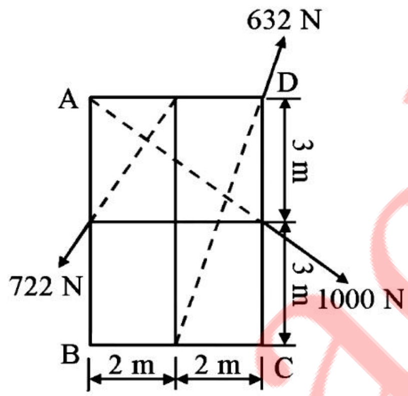
$u = 0$ and $s = 1.6$ m

Substituting the values in equation

$v = 1.5344$ m/s

Velocity of block P_1=1.5344 m/s (towards down)

Q2(a) Compute the resultant of three forces acting on the plate shown in the figure. Locate it's intersection with AB and BC. (6 marks)

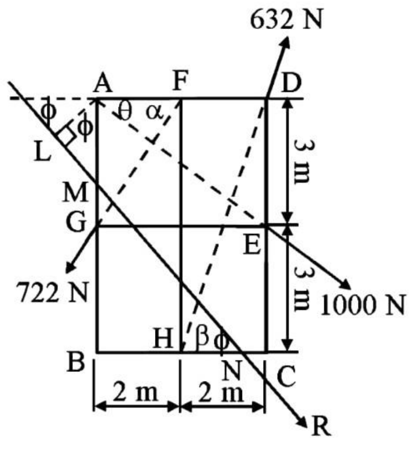


Solution :

Given : Various forces acting on a body

To find : Resultant of the forces and intersection of resultant with AB and BC

Solution :



In $\triangle$ AFG ,

$\tan\alpha = \frac{AG}{AF} = \frac{DE}{BH} = \frac{3}{2} = 1.5$

$\alpha = \tan^{-1}(1.5) = 56.31^o$

In $\triangle$DAE,

$\tan\theta = \frac{DE}{AD} = \frac{DE}{BC} = \frac{3}{4} = 0.75$

$\theta = \tan^{-1}0.75 = 36.87^o$

In $\triangle$DHC

$\tan\beta = \frac{DC}{HC} = \frac{6}{2} = 3$

$\beta = \tan^{-1}(3)$

$\beta = 71.565^o$

Assume R be the resultant of the forces

$\Sigma F_x = -722\cos\alpha + 1000\cos\theta + 632\cos\beta$

$= 599.3624$ N

$\Sigma F_y = -722\sin\alpha - 1000\sin\theta + 632\sin\beta$

$= -601.1725$ N

$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$

$R = \sqrt{(599.3624)^2 + (-601.1725)^2}$

R=848.9073 N

$\phi = \tan^{-1}(\frac{\Sigma Fy}{\Sigma Fx})$

$= \tan^{-1}(\frac{-601.1725}{599.3624})$

$= 45.0863^o$ (in fourth quadrant)

Let R cut AB and BC at points M and N respectively

Draw AL ⊥ R

Taking moments about point A

M_A = 632 sin β x AD -722cos α x AG

= 632 x sin71.565o x 4 – 722cos56.31^o x 3

=1196.7908 Nm

Applying Varigon's theorem

M_A = R x AL

1196.7908 = 848.9073 x AL

AL=1.4098 m

In △AML,

$\cos \Phi = \frac{AL}{AM}$

$\cos 45.0863 = \frac{1.4098}{AM}$

AM = 1.9967 m

MB = AB - AM

= 6 - 1.9967

= 4.0033 m

In △BMN

$\tan \Phi = \frac{BM}{BN}$

$\tan 45.0863 = \frac{4.0033}{BN}$

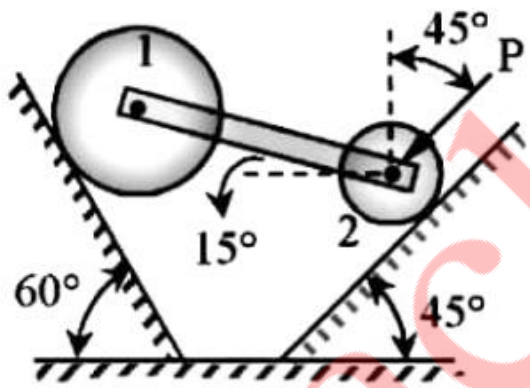
BN = 3.9912 m

R=848.9073 N (45.0863^o in fourth quadrant)

Resultant force intersects AB and BC at M and N such that AM=1.9967 m and BN=3.9912 m

Q.2(b) Two cylinders 1 and 2 are connected by a rigid bar of negligible weight hinged to each cylinder and left to rest in equilibrium in the position shown under the application of force P applied at the center of cylinder 2.

Determine the magnitude of force P.If the weights of the cylinders 1 and 2 are 100N and 50 N respectively. (8 marks)



Solution :

Given : $W_1 = 100$ N

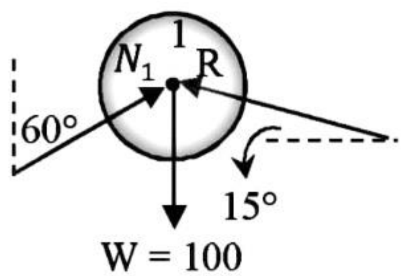
$W_2 = 50$ N

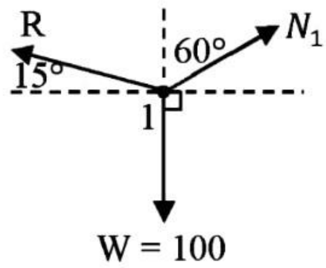
Cylinders are connected by a rigid bar

To find : Magnitude of force P

Solution :

Consider cylinder I





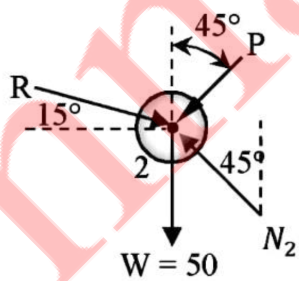
Applying Lami's theorem :

$$\frac{R}{\sin(90+30)} = \frac{W}{\sin(60+75)} = \frac{N_1}{\sin(90+15)}$$

$$R = \frac{100}{\sin 135} \text{ x } \sin 120$$

R = 122.4745 N

Cylinder 2 is under equilibrium



Applying conditions of equilibrium

$\Sigma Fy = 0$

$N_2\sin45 - R\sin15 - P\sin45 – W = 0$

$N_2\sin45 - P\sin45 = 122.4745 \text{ x } 0.2588 + 50$

$N_2\sin45 - P\sin45 = 81.6987$ ………..(1)

Applying conditions of equilibrium

$\Sigma Fx = 0$

$-N_2\cos45+R\cos15-P\cos45=0$

$N_2\cos45+P\cos 45=118.3013$ ……(2)

Solving (1) and (2)

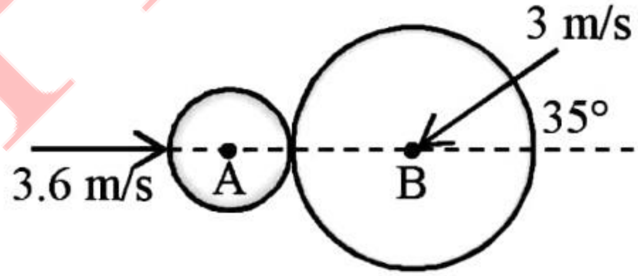
P=25.8819 N

Magnitude of force P required = 25.8819 N

Q.2(c) Just before they collide,two disk on a horizontal surface have velocities shown In figure.

Knowing that 90 N disk A rebounds to the left with a velocity of 1.8 m/s.Determine the rebound velocity of the 135 N disk B.Assume the impact is perfectly elastic.

(6 marks)



Solution :

Given : $W_A = 90N$

$W_B = 135\ N$

Taking velocity direction towards right as positive and towards left as negative

Initial velocity of disk A= 3.6 m/s

Final velocity of disk A=-1.8 m/s

Initial velocity of disk B= 3 m/s

To find : Rebound velocity of disk B

Solution :

$m_A = \frac{90}{g}$ kg

$m_B = \frac{135}{g}$ kg

Consider the X and Y components of u_B

$u_{BX} = -u_B\cos 35 = -2.4575$ m/s

$u_{BY} = -u_B\sin 35 = -1.7207$ m/s

APPLYING LAW OF CONSERVATION OF MOMENTUM :

$m_A u_A + m_B u_B = m_A v_A + m_B v_B$

$\frac{90}{g} \times 3.6 + \frac{135}{g} \times (-2.4575) = \frac{90}{g} \times (-1.8) + \frac{135}{g} \times v_{BX}$

$v_{BX} = 1.1425$ m/s

As the impact takes place along X-axis,the velocities of two disks remains same along Y-axis

$v_{BY} = u_{BY} = -1.7207$ m/s

$v = \sqrt{(v_{BX})^2 + (v_{BY})^2}$

$v = \sqrt{1.1425^2 + (-1.7207)^2}$

$v = 2.0655$ m/s

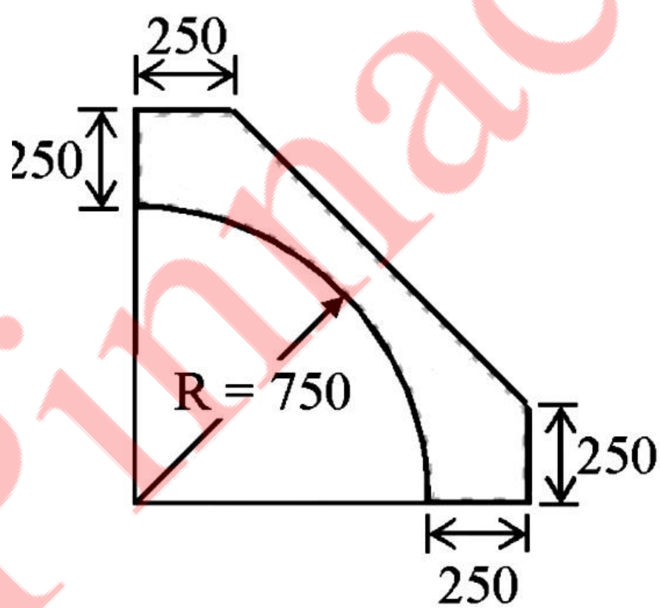
$\alpha = \tan^{-1}(\frac{-1.7207}{1.1425})$

α = 56.4169º

VELOCITY OF DISK B AFTER IMPACT = 2.0655 m/s (56.4169o in fourth quadrant)

Q.3(a) Find the centroid of the shaded portion of the plate shown in the figure.

(8 marks)



Solution :

Y = X is the axis of symmetry

The centroid would lie on this line

Sr.no.	PART	AREA(in mm2)	X co-ordinate(mm)	Ax(mm3)

1.	RECTANGLE	=1000 X 1000 =1000000	$\frac{1000}{2}$ = 500	500000000
2.	TRIANGLE (to be removed)	$\frac{1}{2}$ X 750 X 750 = -281250	$1000 - \frac{750}{3}$ = 750	-210937500
3.	QUARTER CIRCLE (To be removed)	$\frac{\pi r^2}{4}$ = 441786.4669	$\frac{4 X 750}{3\pi}$ = 3141.5926	-140625000
	TOTAL	276963.4669		148437500

$\bar{X} = \frac{\Sigma Ax}{\Sigma A} = \frac{148437500}{276963.5331}$ = **535.946 mm**

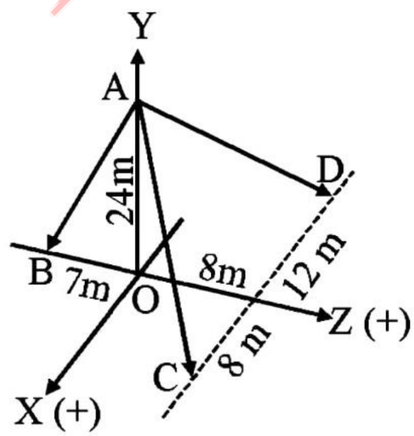
$\bar{y} = \bar{X}$ = **535.946 mm**

CENTROID IS AT (535.946,535.946)mm

Q.3(b) Co-ordinate distance are in m units for the space frame in figure.

There are 3 members AB,AC and AD.There is a force W=10 kN acting at A in a vertically upward direction.

Determine the tension in AB,AC and AD. (6 marks)



Solution :

Given : A = (0,24,0)

B = (0,0,-7)

C = (8,0,8)

D = (-12,0,8)

To find : Tension in AB,AC and AD.

Solution :

Assume $\bar{a},\bar{b},\bar{c},\bar{d}$ be the position vectors of points A,B,C,D with respect to origin O.

$\overline{OA} = \bar{a} = 24\bar{j}$

$\overline{OB} = \bar{b} = -7\bar{k}$

$\overline{OC} = \bar{c} = 8\bar{i} + 8\bar{k}$

$\overline{OD} = \bar{d} = -12\bar{i} + 8\bar{k}$

$\overline{AB} = \bar{b} - \bar{a} = -24\bar{j} - 7\bar{k}$ Magnitude = 25 Unit vector = $\frac{-24j-7k}{25}$

$\overline{AC} = \bar{c} - \bar{a} = 8(\bar{i} - 3\bar{j} + \bar{k})$ Magnitude= $8\sqrt{11}$ Unit vector = $\frac{8(i-3j+k)}{8\sqrt{11}}$

$\overline{AD} = \bar{d} - \bar{a} = 4(-3\bar{i} - 6\bar{j} + 2\bar{k})$ Magnitude = 28 Unit vector = $\frac{4(-3i-6j+2k)}{28}$

Assume T_1,T_2 and T_3 be the tensions along AB,AC and AD

$\mathbf{T_1 = T_1(\frac{-24j-7k}{25})}$

$\mathbf{T_2 = T_2(\frac{8(i-3j+k)}{8\sqrt{11}})}$

$\mathbf{T_3 = T_3(\frac{4(-3i-6j+2k)}{28})}$

A force of 10kN is acting at point A in vertically upward direction

Applying conditions of equilibrium

$\mathbf{10\bar{j} + T_1 + T_2 + T_3 = 0}$

$-10\bar{j} = T_1(\frac{-24j-7k}{25}) + T_2(\frac{8(i-3j+k)}{8\sqrt{11}}) + T_3(\frac{4(-3i-6j+2k)}{28})$

$0\bar{\imath} - 10\bar{j} + 0\bar{k} = T_1(\frac{-24j-7k}{25}) + T_2(\frac{8(i-3j+k)}{8\sqrt{11}}) + T_3(\frac{4(-3i-6j+2k)}{28})$

Comparing both sides of equation

$\frac{T2}{\sqrt{11}} - \frac{3T_3}{7} = 0$

$\frac{-24T_1}{25} - \frac{3T_2}{\sqrt{11}} - \frac{6T_3}{7} = -10$

$\frac{-7T_1}{25} \frac{T_2}{\sqrt{11}} + \frac{2T_3}{7} = 0$

Solving the equations simultaneously

T_1=5.5556 N

T_2=3.0955 N

T_3=2.1778

$T_{AB} = -5.3333\,\bar{j} - 1.5556\bar{k}$

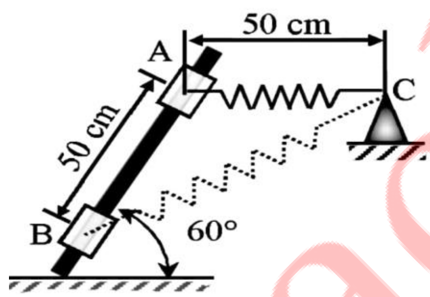
$T_{AC} = 0.9333\bar{\imath} - 2.8\,\bar{j} + 0.9333\bar{k}$

$T_{AD} = -0.9333\bar{\imath} - 1.8667\,\bar{j} + 0.6222\bar{k}$

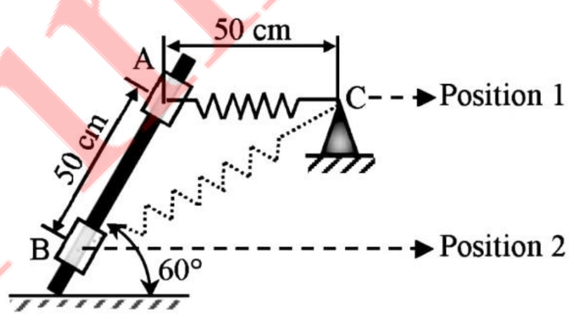
Q.3(c) A 50 N collar slides without friction along a smooth and which is kept inclined at 60° to the horizontal.

The spring attached to the collar and the support C.The spring is unstretched when the roller is at A(AC is horizontal).

Determine the value of spring constant k given that the collar has a velocity of 2.5 m/s when it has moved 0.5 m along the rod as shown in the figure. (6 marks)



Solution :



Given : W=50 N

AB = AC = 0.5 m

To find : Spring constant

Solution :

Mass of collar = $\frac{50}{g}$ kg

Let us assume that h = 0 at position 2

POSITION 1 :

x = 0

$E_{s1} = \frac{1}{2}$ x k x $x_1^2 = 0$

h_1 = 0.5sin60 = 0.433 m

PE1=mgh_1=21.65 J

v_A = 0 m/s

KE_1 = 0J

POSITION II :

v_B = 2.5 m/s

PE_2 = mgh = 0 J (because h=0)

$KE_2 = \frac{1}{2}\ X\ m\ v^2 = \frac{1}{2}\ X\ \frac{50}{g}\ X\ 2.5^2$

= 15.9276 J

In △ABC

Applying cosine rule

$BC^2 = AB^2 + AC^2$ -2 X AB X AC X cos(BAC)

$= 0.5^2 + 0.5^2 - 2$ x 0.5 x 0.5 x cos120

= 0.75

BC = 0.866 m

Un-stretched length of the spring = 0.5 m

Extension of spring(x) = 0.866 - 0.5

=0.366 m

$E_{s2} = \frac{1}{2}$ x k x_2^2

= 0.067k

APPLYING WORK ENERGY PRINCIPLE

$U_{1-2} = KE_2 - KE_1$

$PE_1 - PE_2 + E_{S1} - ES_2 = KE_2 - KE_1$

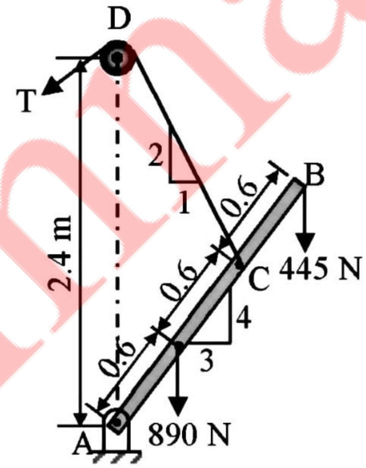
21.6506-0+0-0.067K=15.9276-0

K = 85.4343 N/m

SPRING CONSTANT IS 85.4343 N/m

Q.4(a) A boom AB is supported as shown in the figure by a cable runs from C over a small smooth pulley at D.

Compute the tension T in cable and reaction at A.Neglect the weight of the boom and size of the pulley. (8 marks)



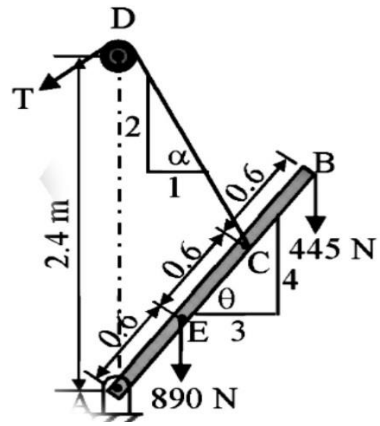
Solution :

Given : Beam AB is supported by a cable

To find : Tension T in cable

Reaction at A

Solution :



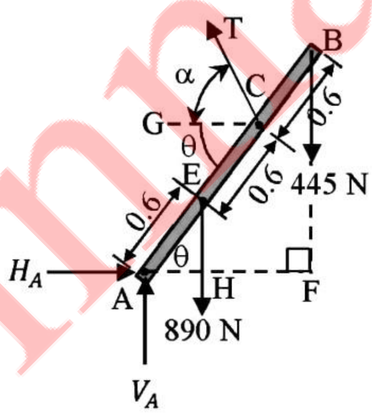
tan α = $\frac{2}{1}$

α = 63.4349°

tan θ = $\frac{4}{3}$

θ = 53.13°

Assume H_A and V_A be the horizontal and vertical reaction forces at A



∠GCA = ∠BAF = θ

∠TCG = α

∠TCA = α + θ

= 63.4349° + 53.16°

= 116.5651°

∠TCB = 180° - 116.5651°

= 63.4349°

AC = AE+EC = 0.6 + 0.6 = 1.2

AB = AC + CB = 1.2 + 0.6 = 1.8

AF = ABcos θ = 1.8cos53.13 = 1.08

AH = AEcos θ = 0.6cos53.13 = 0.36

BEAM AB IS INDER EQUILIBRIUM

Applying conditions of equilibrium

$\Sigma M_A = 0$

-445 X AF – 890 X AH + Tsin63.4349 X AC = 0

T X 0.8944 X 1.2 = 445 X 1.08 + 890 X 0.36

T = 746.2877 N

$\Sigma F_X = 0$

H_A - Tcos63.4349 = 0

H_A=333.75 N

$\Sigma F_Y = 0$

V_A + Tsin63.4349 – 890 – 445 = 0

V_A = 667.5 N

$R_A = \sqrt{H_A^2 + V_A^2}$

$R_A = \sqrt{(333.75)^2 + (667.5)^2}$

R_A = 746.2877 N

$\Phi = \tan^{-1}(\frac{V_A}{H_A})$

$\Phi = \tan^{-1}(\frac{667.5}{333.75})$

Φ =**63.4395°**

Tension in cable = 746.2877 N (63.43949° in second quadrant)
Reaction at A = 746.2877 N (63.4395° in first quadrant)

Q.4(b) The acceleration of the train starting from rest at any instant is given by the expression a=$\frac{8}{v^2+1}$ where v is the velocity of train in m/s.

Find the velocity of the train when its displacement is 20 m and its displacement when velocity is 64.8 kmph. (6 marks)

Solution :

Given : a=$\frac{8}{v^2+1}$

To find : Velocity when displacement is 20 m

Displacement when velocity is 64.8 kmph.

Solution :

a = v$\frac{dv}{dx}$

v$\frac{dv}{dx}$ = $\frac{8}{v^2+1}$

$v(v^2+1)dv = 8dx$

Integrating both sides

$\int v(v^2+1)dv = \int 8dx$

$\frac{v^4}{4} + \frac{v^2}{2} = 8x + c$ ……….(1)

Multiplying by 4 on both sides

$V^4 + 2v^2 = 32x + 4c$

Substituting v=0 and x=0 in (1)

c=0

From (1)

$V^4 + 2v^2 = 32x$ …………..(2)

Case 1 : x=20 m

$V^4 + 2v^2 = 32 \times 20$ ………(From 2)

$V^4 + 2v^2 - 640 = 0$

Solving the equation

$V^2 = 24.3180$

V=4.9361 m/s

Case 2 : V=64.8 kmph(or v = 18 m/s)

$18^4 + 2 \times 18^2 = 32x$ …………..(From 2)

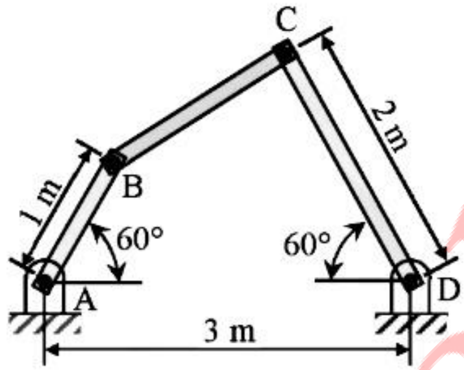
$1.5624 = 32x$

x = 3300.75 m

When displacement of train is 20 m,then velocity is 4.9361 m/s

When velocity of the train is 64.8 kmph,then its displacement is 3300.75m

Q.4(c) Angular velocity of connector BC is 4 r/s in clockwise direction.What is the angular velocities of cranks AB and CD? (6 marks)



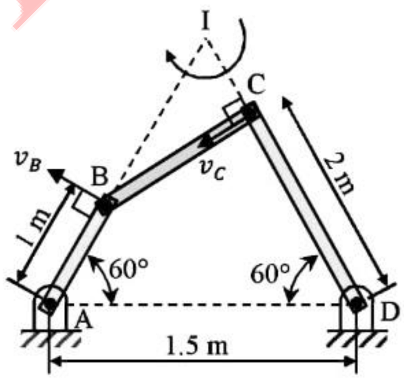
Solution:

Given : Angular velocity of BC is 4 rad/s

To find : Angular velocity of AB and CD

Solution:

ICR is shown in the figure



USING GEOMETRY :

In △IAD

$\angle A = \angle D = 60^o$
$\angle I = 60^o$

$\triangle$ IAD is equilateral

IA = ID = AD = 3 cm

IB + AB = IA

IB = 2 cm

Similarly,we can solve that IC = 1 cm

v = rω

$v_B = IB \times \omega_{BC} = 8$ m/s

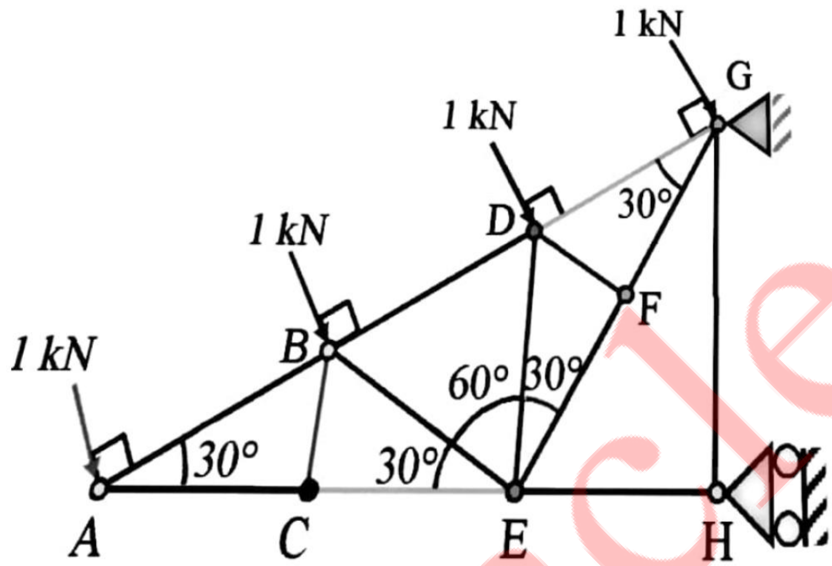
$v_C = IC \times \omega_{BC} = 4$ m/s

$\omega_{AB} = \frac{v_B}{AB} = \frac{8}{1} = 8$ rad/s(Anti-clockwise)

$\omega_{DC} = \frac{v_c}{DC} = \frac{4}{2} = 2$ rad/s(Anti-clockwise)

Angular velocity of AB=8 rad/s(Anti-clockwise)
Angular velocity of CD=2 rad/s(Anti-clockwise)

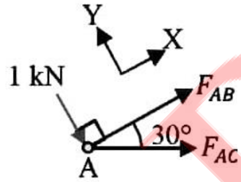
Q.5(a) In the truss shown in figure,compute the forces in each member. (8 marks)



Solution :

We can say that FD,GH and CB are zero force members in the given truss

Joint A :



Applying the conditions of equilibrium

$\Sigma Fy=0$

$-1 - F_{AC} \sin 30 = 0$

$\mathbf{F_{AC} = -2kN}$

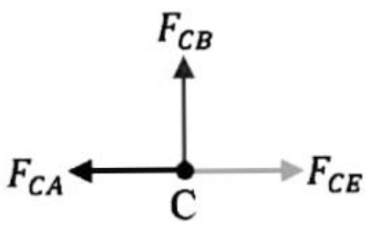
Applying the conditions of equilibrium

$\mathbf{\Sigma Fx = 0}$

$F_{AB} + F_{AC} \cos 30 = 0$

F_{AB} = 1.7321 Kn

JOINT C :

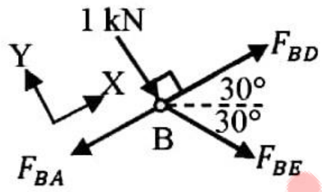


Applying the conditions of equilibrium

$\Sigma Fx = 0$

$F_{CE} = F_{CA}$ = -2kN

JOINT B :



Applying the conditions of equilibrium

$\Sigma Fy = 0$

$-1 - F_{BE} \sin 60 = 0$

FBE = -1.1547 kN

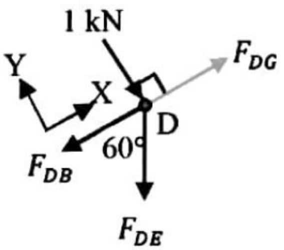
Applying the conditions of equilibrium

$\Sigma Fx = 0$

$-F_{BA} + F_{BE}\cos 60 + F_{BD} = 0$

F_{BD} = 2.3094 kN

JOINT D :



Applying the conditions of equilibrium

$\Sigma Fy = 0$

$-1 - F_{DE}\sin 60 = 0$

$\mathbf{F_{DE} = -1.1547\ kN}$

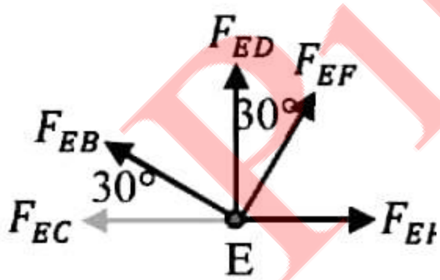
Applying the conditions of equilibrium

$\Sigma Fx = 0$

$-F_{DB} - F_{DE}\cos 60 + F_{DG} = 0$

$\mathbf{F_{DG} = 1.7321\ kN}$

JOINT E :



Applying the conditions of equilibrium

$\Sigma Fy = 0$

$F_{ED} + F_{EF}\cos 30 + F_{EB}\sin 30 = 0$

$F_{EF}\cos 30 = -(-1.1547)-(-1.1547) \times \frac{1}{2}$

$\mathbf{F_{EF} = 2kN}$

Applying the conditions of equilibrium

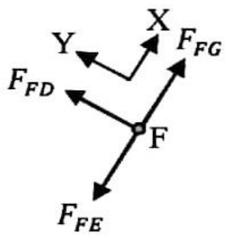
$\Sigma Fx = 0$

$-F_{EC} + F_{EH} + F_{EF}\sin30 - F_{EB}\cos30 = 0$

$F_{EH} = F_{EC} - F_{EF}\sin30 + F_{EB}\cos30$

FEH = -4kN

Joint F :



Applying the conditions of equilibrium

$\Sigma Fx = 0$

$F_{FG} = F_{FE} = -2kN$

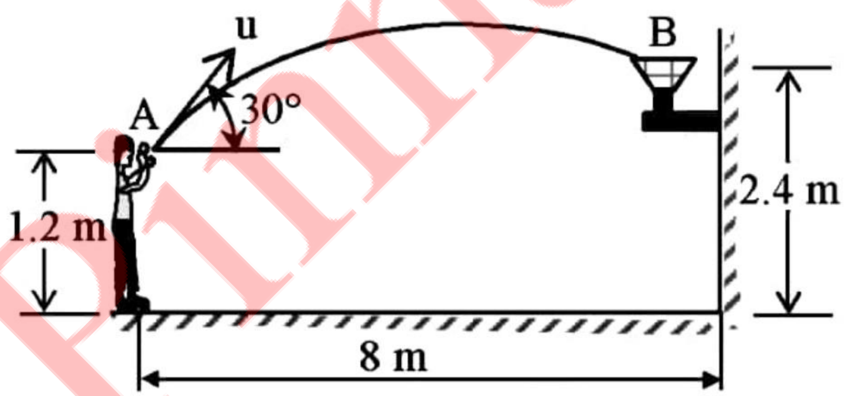
Final answer :

Sr.no.	MEMBER	MAGNITUDE OF FORCE (in kN)	NATURE OF FORCE
1.	AC	2	COMPRESSION
2.	AB	1.7321	TENSION
3.	CB	0	-
4.	CE	2	COMPRESSION
5.	BE	1.1547	COMPRESSION
6.	BD	2.3094	TENSION
7.	DE	1.1547	COMPRESSION

8.	DG	1.7321	TENSION
9.	EF	2	TENSION
10.	EH	4	COMPRESSION
11.	FD	0	-
12	FG	2	COMPRESSION
13.	GH	0	-

Q.5(b) Determine the speed at which the basket ball at A must be thrown at an angle 30^o so that if makes it to the basket at B.

Also find at what speed it passes through the hoop. **(6 marks)**



Solution :

Given : $\theta=30^o$

To find : Speed at which basket ball must be thrown

Solution :

Assume that the basket ball be thrown with initial velocity u and it takes time t to reach B

HORIZONTAL MOTION

Here the velocity is constant

8 = ucos30 x t

$t = \frac{8}{ucos30} = \frac{9.2376}{u}$ ……..(1)

v_B = ucos30 (Since velocity is constant in horizontal motion) ………(2)

VERTICAL MOTION

Initial vertical velocity (u_v) = usin30 =0.5u …….(3)

Vertical displacement(s) = 2.4 - 1.2 = 1.2

$t = \frac{9.2376}{u}$

Using kinematical equation :

$s = ut + \frac{1}{2} \times at^2$

$1.2 = \frac{u}{2} \times \frac{9.2376}{u} - \frac{1}{2} \times 9.81 \times (\frac{9.2376}{u})^2$

u^2=122.4289

u=11.0648 m/s

u_v=0.5u (From 3)

u_v = 0.5 x 11.0648

= 5.5324 m/s

Using kinematical equation

$v_v^2 = u_v^2 + 2as$

$v_v^2 = 5.5324^2 - 2 \times 9.81 \times 1.2$

v_v = 2.6622 m/s

v_h = 11.0648cos30 = 9.5824 m/s (From 2)

$v_B = \sqrt{v_v^2 + v_h^2}$

v_B = 9.9441 m/s

$\alpha = \tan^{-1}(\frac{2.6577}{9.5824})$

$= 15.5015^o$

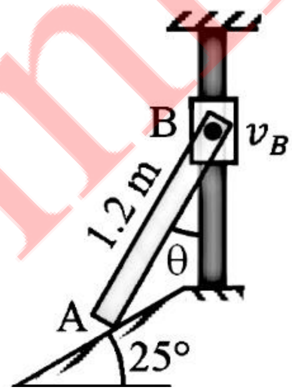
Speed at which the basket-ball at A must be thrown = 11.0648 m/s (30^o in first quadrant)

Speed at which the basket-ball passes through the hoop = 9.9441 m/s(15.5015^o in fourth quadrant)

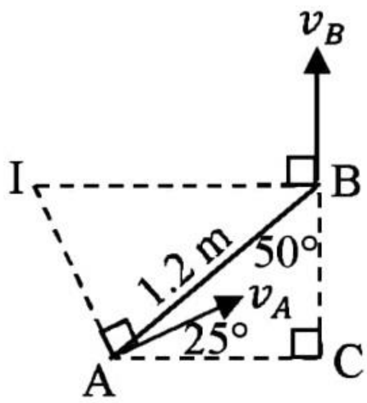
Q.5(c) Figure shows a collar B which moves upwards with constant velocity of 1.5 m/s.At the instant when $\theta=50^o$.Determine :

(i)The angular velocity of rod pinned at B and freely resting at A against 25o sloping ground.

(ii)The velocity of end A of the rod. (6 marks)



Solution:



ICR is shown in the given figure

BY USING GEOMETRY:

In △ABC

∠ABC = 50

∠ACB = 90

∠BAC = 40

∠CAV = 25

∠BAV = 40 – 25 = 15

IA ⊥ V_A

∠IAB = 90 – 15 = 75

∠IBA = 90 – 50 = 40

In △IBA

∠BIA = 180 – 75 = 65

In △IBA

AB=1.2 m

APPLYING SINE RULE

$$\frac{AB}{\sin I} = \frac{IB}{\sin A} = \frac{IA}{\sin B}$$

$$\frac{1.2}{\sin 65} = \frac{IB}{\sin 75} = \frac{IA}{\sin 40}$$

IB=1.2789 m

IA=0.8511 m

Assume ω_{AB} be the angular velocity of AB

$\omega_{AB} = \frac{v_B}{r} = \frac{v_B}{IB} = \frac{1.5}{1.2789} = 1.1728$ rad/s

v_A = r x AB = IA x ω_{AB} = 0.8511 x 1.7288 = 0.99825 m/s

Angular velocity of rod AB= 1.1728 rads (Anti-clockwise)

Instantaneous velocity of A = 0.9982 m/s(25° in first quadrant)

Q.6(a) A force of 140 kN passes through point C (-6,2,2) and goes to point B (6,6,8). Calculate moment of force about origin. (4 marks)

Solution :

Given : C (-6,2,2)

B (6,6,8)

To find : Moment of force about origin

Solution :

Assume $\bar{b}$ and $\bar{c}$ be the position vectors of points B and C respectively w.r.t O (0,0,0)

$\overline{OB} = \bar{b} = 6\bar{\imath} + 6\bar{\jmath} + 8\bar{k}$

$\overline{OC} = -6\bar{\imath} + 2\bar{\jmath} + 2\bar{k}$

$\overline{CB} = (6\,\bar{\imath} + 6\bar{\jmath} + 8\bar{k}) - (-6\,\bar{\imath} + 2\bar{\jmath} + \bar{k})$

$= 2\,(6\,\bar{\imath} + 2\bar{\jmath} + 3\bar{k})$

$|\overline{CB}| = 2 \times \sqrt{6^2 + 2^2 + 3^2}$

$= 14$

Unit vector along $\overline{CB} = \frac{CB}{|CB|} = \frac{6i+2j+3k}{7}$

Force along $\overline{CB} = \bar{F} = 140 \times \frac{6i+2j+3k}{7}$

$= 120\,\bar{\imath} + 40\bar{\jmath} + 60\bar{k}$

Moment of $\bar{F}$ about O = $\overline{OB} \times \bar{F}$

i	j	k
6	6	8
120	40	60

$= 40\,\bar{\imath} + 600\bar{\jmath} - 480k$

Moment of F about C is $40\,\bar{\imath} + 600\bar{\jmath} - 480\bar{k}$ kNm

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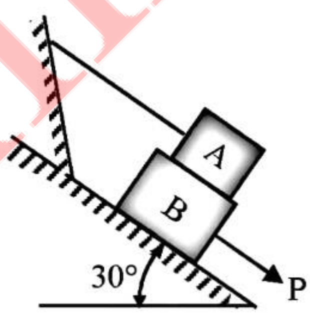
Q.6(b) Refer to figure.If the co-efficient of friction is 0.60 for all contact surfaces and θ = 30°,what force P applied to the block B acting down and parallel to the incline will start motion and what will be the tension in the cord parallel to inclined plane attached to A.

Take W_A=120 N and W_B=200 N. (8 marks)



Solution :

Given : : μ=0.6

θ = 30°

W_A = 120 N

$W_B = 200$ N

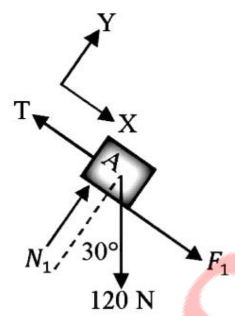
To find : Force P

Solution :

$F_1 = \mu N_1 = 0.6N_1$ ………(1)

$F_2 = \mu N_2 = 0.6N_2$ ……….(2)

Consider FBD of block A



The block is considered to be in equilibrium

Applying conditions of equilibrium

$\Sigma Fy = 0$

$N_1 - 120\cos 30 = 0$

N_1 = 103.923 N ……….(3)

From (1)

$F_1 = 0.6 \times 103.923$

$= 62.3538$ N

Applying conditions of equilibrium

$\Sigma Fx = 0$

$F_1 + 120\sin 30 - T = 0$

T = 122.3538 N

Consider FBD of block B

Applying conditions of equilibrium

$\Sigma Fy = 0$

$N_2 - N_1 - 200\cos 30 = 0$

$\mathbf{N_2 = 277.1281\ N}$

$F_2 = 0.6 \times 277.1281$

$= 166.2769$ N From (2)

Applying conditions of equilibrium

$\Sigma Fx = 0$

$P - F_1 - F_2 + 200\sin 30 = 0$

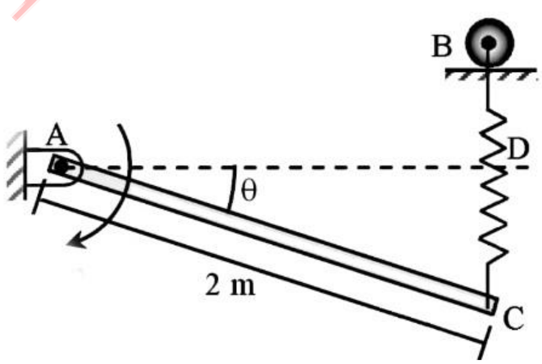
$\mathbf{P = 128.6307\ N}$

Force required on block B to start the motion is 128.6307 N

Tension T in the cord parallel to inclined plane attached to A=122.3538 N

Q.6(c) Determine the required stiffness k so that the uniform 7 kg bar AC is in equilibrium when $\theta=30^o$.

Due to the collar guide at B the spring remains vertical and is unstretched when $\theta = 0^o$.Use principle of virtual work. (4 marks)



Solution:

Given : : Mass of bar AC = 7 kg

$\theta = 30^o$

To find : Required stiffness k

Solution:

Weight of rod = 7g N

Assume rod AC have a small virtual angular displacement $\delta\theta$ in anti-clockwise direction

Reaction forces H_A and V_A do not do any virtual work

Un-stretched length of the spring = BD

Extension of the spring (x) = CD = 2sin θ

Assume F_S be the spring force at end C of the rod

$F_S = Kx = 2K\sin\theta$

Assume A to be the origin and AD be the X-axis of the system

Active force	Co-ordinate of the point of action along the force	Virtual Displacement
W=7g	-sin θ	$\delta yM = -\cos\theta\,\delta\theta$
FS=2Ksin θ	-2sin θ	$\delta yC' = -2\cos\theta\,\delta\theta$

APPLYING PRINCIPLE OF VIRTUAL WORK

$\delta U = 0$

$-W \times \delta_{YM} + F_S \times \delta Y_{C'} + 50 \times \delta\theta = 0$

$2K\sin\theta \times (-2\cos\theta\,\delta\theta) = 7g \times (-\cos\theta\,\delta\theta) - 50 \times \delta\theta$

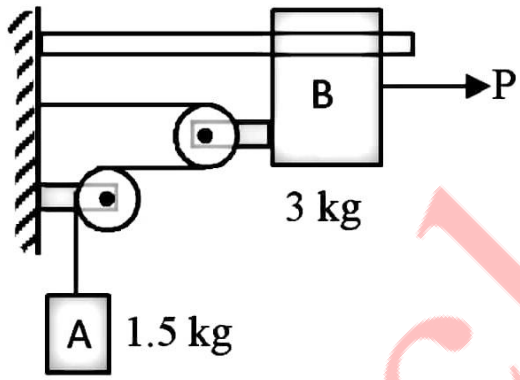
Substituting the value of θ and solving

K=63.2025 Nm

The required stiffness K for bar AC to remain in equilibrium is 63.2025 Nm

Q.6(d) The system in figure is initially at rest.

Neglecting friction determine the force P required if the velocity of the collar is 5 m/s after 2 sec and corresponding tension in the cable. (4 marks)



Solution :

For block B

u = 0

t = 2 s

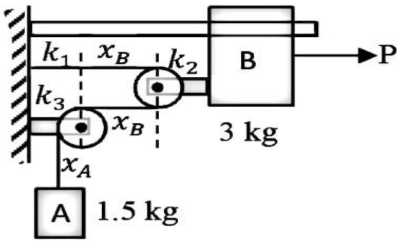
v = 5m/s

$\mathbf{a = \frac{5-0}{2} = 2.5\ m/s^2}$ …………..(1)

Assume the string across the two pulleys be of length L

Assume x_A and x_B be the displacements of block A and collar B respectively

Assume k_1,k_2 and k_3 be the lengths of the string which remain constant irrespective of the position of block A and block B



$k_1 + x_B + k_2 + x_B + k_3 + x_A = L$

$\mathbf{x_A = L - k_1 - k_2 - k_3 - 2x_B}$

Differentiating with respect to time

$\mathbf{v_A = -2v_B}$

Differentiating with respect to time one again

$\mathbf{a_A = -2a_B}$

Considering only magnitude

$\mathbf{a_A = 2a_B}$

$a_A = 2 \times 2.5$

$= 5\ m/s^2$ ………..(2) (From 1)

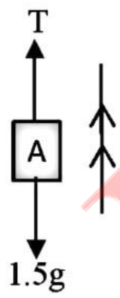
Weight of block A(WA) = $m_A g$

= 14.715 N

Assume T to be the tension in the string

Consider the vertical motion of block A

F.B.D of block A



$\Sigma Fy = m_A a_A$

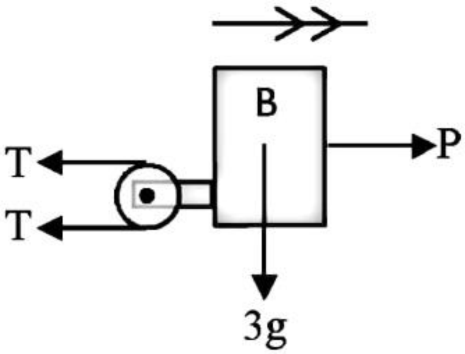
$T – W_A = m_A a_A$

$T - 14.715 = 1.5 \times 5$

T = 22.215 N …………..(3)

Consider the horizontal motion of collar B

F.B.D of collar B



$\Sigma F_x = m_B a_B$

$P - 2T = m_B a_B$

$P - 2 \times 22.215 = 3 \times 2.5$

$P = 51.93$ N

Force P required = 51.93 N
Tension in the cable = 22.215 N